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Counterposition and negative phase velocity in uniformly moving dissipative materials

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Abstract

We considered the phenomena of counterposition and negative phase velocity, which are relevant to certain metamaterials and certain astrophysical scenarios. The Lorentz transformations of electric and magnetic fields were implemented to study (i) the refraction of linearly polarized plane waves into a half-space occupied by a uniformly moving material and (ii) the traversal of linearly polarized Gaussian beams through a uniformly moving slab. Motion was taken to occur tangentially to the interface(s) and in the plane of incidence. The moving materials were assumed to be isotropic, homogeneous and dissipative dielectric materials from the perspective of a co-moving observer. Two different moving materials were considered: from the perspective of a co-moving observer, material A supports planewave propagation with only positive phase velocity, whereas material B supports planewave propagation with both positive and negative phase velocity, depending on the polarization state. For both materials A and B, the sense of the phase velocity and whether or not counterposition occurred, as perceived by a non-co-moving observer, could be altered by varying the observer's velocity. Furthermore, the lateral position of a beam upon propagating through a uniformly moving slab made of material A, as perceived by a non-co-moving observer, could be controlled by varying the observer's velocity. In particular, at certain velocities, the transmitted beam emerged from the slab laterally displaced in the direction opposite to the direction of incident beam. The transmittances of a uniformly moving slab made of material B were very small and the energy density of the transmitted beam was largely concentrated in the direction normal to the slab, regardless of the observer's velocity.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

What happens to plane waves at the planar interface of two passive linear mediums is one of the staple problems of electrodynamics. Relative motion between the two mediums on either side of the interface can give rise to complex electromagnetic behaviour, even when both mediums are simple isotropic dielectric-magnetic mediums with respect to a co-moving observer. This complex behaviour may encompass negative refraction [1] and the allied phenomena of *counterposition* and *negative phase velocity* (NPV). These phenomena have come under increasing scrutiny recently, partly because they also arise in certain exotic metamaterials (in non-relativistic scenarios). Accordingly, the elucidation of counterposition and NPV—which is the chief goal of this paper—has significance for astrophysical applications as well as the design of novel materials.

We investigate the effects of relative uniform motion on the phenomena of counterposition and NPV. The orientation of the cycle-averaged Poynting vector is central to both phenomena. The phase velocity is said to be negative (positive) if it casts a negative (positive) projection onto the cycle-averaged Poynting vector. NPV has commonly been understood to betoken negative refraction [2, 3]. However, certain anisotropic mediums can support NPV propagation in conjunction with positive refraction (and vice versa) [4], and we recently demonstrated that the correspondence between NPV and negative refraction is only strictly appropriate in the case of uniform planewave propagation in isotropic dielectric-magnetic mediums [5]. Usually, at a planar interface, the cycle-averaged Poynting vector and the real part of the wave vector associated with the refracted planewave are oriented on the same side of the normal to the planar interface [6]. But under certain conditions, these two vectors can be counterposed, i.e., oriented on opposite sides of the normal to the planar interface. This counterposition has been described in certain isotropic dielectric [5] and anisotropic mediums [4, 7, 8] in nonrelativistic scenarios, as well as in certain relativistic scenarios³ [9, 10].

Previous theoretical studies concerning counterposition and NPV in uniformly moving mediums have mostly relied upon the Minkowski constitutive relations to describe the moving medium in a non-co-moving inertial reference frame [9–11], following the standard textbook approach [6]. These studies have shown that nondissipative mediums which do not support NPV and counterposition from the perspective of a co-moving observer may support NPV from the perspective of certain non-co-moving inertial reference frames. Furthermore, by considering the propagation of a Gaussian beam through a uniformly moving, nondissipative slab, we found that a degree of concealment could be achieved under certain conditions [11]. However, the Minkowski constitutive relations are strictly appropriate only for instantaneously responding mediums [12]. For realistic material mediums, it is necessary to incorporate both spatial and temporal nonlocality in the non-co-moving inertial reference frame.

Recently, using a direct approach in which the Lorentz transformations of the electric and magnetic fields are implemented, a dissipative medium which does not support NPV from the perspective of a co-moving observer was observed to support NPV from the perspective of certain non-co-moving inertial reference frames [13]. This direct approach—which does not involve the Minkowski constitutive relations—is implemented in the following sections to investigate whether counterposition and NPV can be induced in uniformly moving, realistic materials and, if so, under what circumstances. A particular focus of our attention is the behaviour of an isotropic dielectric material which supports the propagation of nonuniform

³ We note that the phenomenon of counterposition is referred to as ‘negative refraction’ in [9]. In contrast, and in keeping with the law of conservation of linear momentum parallel to the interface, we adopt the standard convention wherein the sense of refraction is determined solely by the relative orientations of the real parts of the refraction and incidence wave vectors.

plane waves with both positive and negative phase velocity, depending upon the polarization state, in a co-moving inertial reference frame. This unexpected property for an isotropic dielectric material came to light only very recently [5].

As regards notation 3-vectors are in boldface with the $\hat{}$ symbol denoting a unit vector; and the identity 3×3 dyadic is expressed as $\underline{\underline{I}} = \hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} + \hat{\mathbf{z}}\hat{\mathbf{z}}$. The operators Re and Im yield the real and imaginary parts of complex quantities; and $i = \sqrt{-1}$. The permittivity and permeability of free space are written as ϵ_0 and μ_0 , respectively; and $c_0 = 1/\sqrt{\epsilon_0\mu_0}$ is the speed of light in free space.

2. Refraction into a moving half-space

2.1. Theoretical considerations

Let us consider a homogeneous material which is a spatially local, isotropic, dielectric material, characterized by the frequency-domain constitutive relations

$$\left. \begin{aligned} \mathbf{D}' &= \epsilon_0 \epsilon' \mathbf{E}' \\ \mathbf{B}' &= \mu_0 \mathbf{H}' \end{aligned} \right\}, \quad (1)$$

in an inertial reference frame Σ' . Since the material is generally dissipative, the relative permittivity ϵ' is a complex-valued function of the angular frequency ω' . This material occupies the half-space $z > 0$, whereas the half-space $z < 0$ is vacuous.

The inertial reference frame Σ' moves at constant velocity $\mathbf{v} = v\hat{\mathbf{v}}$ relative to the inertial reference frame Σ . We choose $\hat{\mathbf{v}} = \hat{\mathbf{x}}$. The electromagnetic field phasors in Σ are related to those in Σ' by the Lorentz transformations [6]

$$\left. \begin{aligned} \mathbf{E} &= (\mathbf{E}' \cdot \hat{\mathbf{v}}) \hat{\mathbf{v}} + \gamma \left[(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}) \cdot \mathbf{E}' - \mathbf{v} \times \mathbf{B}' \right] \\ \mathbf{B} &= (\mathbf{B}' \cdot \hat{\mathbf{v}}) \hat{\mathbf{v}} + \gamma \left[(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}) \cdot \mathbf{B}' + \frac{\mathbf{v} \times \mathbf{E}'}{c_0^2} \right] \\ \mathbf{H} &= (\mathbf{H}' \cdot \hat{\mathbf{v}}) \hat{\mathbf{v}} + \gamma \left[(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}) \cdot \mathbf{H}' + \mathbf{v} \times \mathbf{D}' \right] \\ \mathbf{D} &= (\mathbf{D}' \cdot \hat{\mathbf{v}}) \hat{\mathbf{v}} + \gamma \left[(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}) \cdot \mathbf{D}' - \frac{\mathbf{v} \times \mathbf{H}'}{c_0^2} \right] \end{aligned} \right\}, \quad (2)$$

where the scalars are

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c_0}. \quad (3)$$

We consider a plane wave described in Σ by the electric and magnetic field phasors

$$\left. \begin{aligned} \mathbf{E}_i &= \mathbf{e}_i \exp [i (\mathbf{k}_i \cdot \mathbf{r} - \omega t)] \\ \mathbf{H}_i &= \mathbf{h}_i \exp [i (\mathbf{k}_i \cdot \mathbf{r} - \omega t)] \end{aligned} \right\}, \quad z \leq 0, \quad (4)$$

where the wave vector

$$\mathbf{k}_i = k_0 (\sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{z}}), \quad (5)$$

with the free-space wave number $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ and ω being the angular frequency with respect to Σ . As the incident plane wave transports energy towards the interface, the angle $\theta \in (-90^\circ, 90^\circ)$ and hence the real-valued scalar

$$\kappa = k_0 \sin \theta \in (-k_0, k_0). \quad (6)$$

Thus, we have chosen the velocity of the moving material as tangential to the interface plane $z = 0$ and lying in the plane of incidence (i.e., the xz -plane).

In Σ' , the counterparts of the phasors (4) are

$$\left. \begin{aligned} \mathbf{E}'_i &= \mathbf{e}'_i \exp[i(\mathbf{k}'_i \cdot \mathbf{r}' - \omega' t')] \\ \mathbf{H}'_i &= \mathbf{h}'_i \exp[i(\mathbf{k}'_i \cdot \mathbf{r}' - \omega' t')] \end{aligned} \right\}, \quad z \leq 0, \quad (7)$$

wherein the phasor amplitudes $\{\mathbf{e}'_i, \mathbf{h}'_i\}$ are related to $\{\mathbf{e}_i, \mathbf{h}_i\}$ via the Lorentz transformations (2), while

$$\left. \begin{aligned} \mathbf{k}_i &= \gamma \left(\mathbf{k}'_i \cdot \hat{\mathbf{v}} + \frac{\omega' v}{c_0^2} \right) \hat{\mathbf{v}} + \left(\underline{\underline{I}} - \hat{\mathbf{v}} \hat{\mathbf{v}} \right) \cdot \mathbf{k}'_i \\ \mathbf{r} &= \left[\underline{\underline{I}} + (\gamma - 1) \hat{\mathbf{v}} \hat{\mathbf{v}} \right] \cdot \mathbf{r}' + \gamma \mathbf{v} t' \\ \omega &= \gamma (\omega' + \mathbf{k}'_i \cdot \mathbf{v}) \\ t &= \gamma \left(t' + \frac{\mathbf{v} \cdot \mathbf{r}'}{c_0^2} \right) \end{aligned} \right\}. \quad (8)$$

The plane wave refracted into the half-space $z > 0$ is represented in Σ' by the electric and magnetic field phasors

$$\left. \begin{aligned} \mathbf{E}'_t &= \mathbf{e}'_t \exp[i(\mathbf{k}'_t \cdot \mathbf{r}' - \omega' t')] \\ \mathbf{H}'_t &= \mathbf{h}'_t \exp[i(\mathbf{k}'_t \cdot \mathbf{r}' - \omega' t')] \end{aligned} \right\}, \quad z \geq 0, \quad (9)$$

wherein the wave vector

$$\mathbf{k}'_t = (\mathbf{k}'_t \cdot \hat{\mathbf{x}}) \hat{\mathbf{x}} + k'_{zt} \hat{\mathbf{z}} \quad (10)$$

adheres to the law of conservation of linear momentum parallel to the interface, the discovery of this law being variously assigned to Ibn Sahl, Thomas Harriot, Willebrord van Snel van Royen and René Descartes [14–16]. Use of the constitutive relations (1) and application of the Maxwell curl postulates in Σ' provides an expression for k'_{zt} and relationships between \mathbf{e}'_t and \mathbf{h}'_t [6].

With respect to Σ' , the reflected plane wave may be expressed as

$$\left. \begin{aligned} \mathbf{E}'_r &= \mathbf{e}'_r \exp[i(\mathbf{k}'_r \cdot \mathbf{r}' - \omega' t')] \\ \mathbf{H}'_r &= \mathbf{h}'_r \exp[i(\mathbf{k}'_r \cdot \mathbf{r}' - \omega' t')] \end{aligned} \right\}, \quad z \leq 0, \quad (11)$$

where

$$\mathbf{k}'_r = (\mathbf{k}'_r \cdot \hat{\mathbf{x}}) \hat{\mathbf{x}} + k'_{zr} \hat{\mathbf{z}}. \quad (12)$$

Application of the Maxwell curl postulates in Σ' provides an expression for k'_{zr} and relationships between \mathbf{e}'_r and \mathbf{h}'_r .

At this point, the usual boundary conditions can be invoked across the plane $z = 0$, i.e.,

$$\left. \begin{aligned} (\mathbf{e}'_t + \mathbf{e}'_r) \cdot \hat{\mathbf{x}} &= \mathbf{e}'_t \cdot \hat{\mathbf{x}} & (\mathbf{e}'_t + \mathbf{e}'_r) \cdot \hat{\mathbf{y}} &= \mathbf{e}'_t \cdot \hat{\mathbf{y}} \\ (\mathbf{h}'_t + \mathbf{h}'_r) \cdot \hat{\mathbf{x}} &= \mathbf{h}'_t \cdot \hat{\mathbf{x}} & (\mathbf{h}'_t + \mathbf{h}'_r) \cdot \hat{\mathbf{y}} &= \mathbf{h}'_t \cdot \hat{\mathbf{y}} \end{aligned} \right\}, \quad z = 0. \quad (13)$$

In this way, the phasor amplitudes $\{\mathbf{e}'_r, \mathbf{h}'_r\}$ and $\{\mathbf{e}'_t, \mathbf{h}'_t\}$ can be determined. Thereafter, the reflected and refracted plane waves can be transformed back to Σ . Most importantly in the present context, the refracted plane wave turns out to be represented in Σ by the electric and magnetic field phasors

$$\left. \begin{aligned} \mathbf{E}_t &= \mathbf{e}_t \exp[i(\mathbf{k}_t \cdot \mathbf{r} - \omega t)] \\ \mathbf{H}_t &= \mathbf{h}_t \exp[i(\mathbf{k}_t \cdot \mathbf{r} - \omega t)] \end{aligned} \right\}, \quad z \geq 0, \quad (14)$$

wherein the wave vector

$$\mathbf{k}_t = \kappa \hat{\mathbf{x}} + k_{zt} \hat{\mathbf{z}}. \quad (15)$$

Since $k_{zt} \in \mathbb{C}$, the refracted plane wave is generally nonuniform.

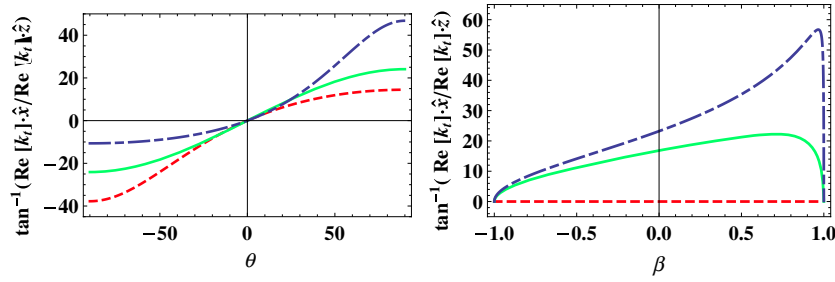


Figure 1. The orientation angle of $\text{Re } \mathbf{k}_t$ (in degree) plotted against angle of incidence $\theta \in (-90^\circ, 90^\circ)$ for $\beta = -0.5$ (broken curve, red), $\beta = 0$ (solid curve, green) and $\beta = 0.7$ (broken dashed curve, blue); and plotted against relative speed $\beta \in (-1, 1)$ for $\theta = 0^\circ$ (broken curve, red), $\theta = 45^\circ$ (solid curve, green) and $\theta = 75^\circ$ (broken dashed curve, blue). The refracting half-space is occupied by material A ($\epsilon' = 6 + 0.05i$). (Colour online.)

2.2. Numerical studies

For the purpose of illustration, let us suppose firstly that the material occupying the half-space $z > 0$ has a relative permittivity $\epsilon' = 6 + 0.05i$ at angular frequency ω' , with respect to the inertial reference frame Σ' .⁴ We call this material A. We note that the case for the corresponding nondissipative material, specified by a real-valued relative permittivity of 6, was investigated previously using an approach based on the Minkowski constitutive relations [11]. The orientation angle of the real part of \mathbf{k}_t , as defined by $\tan^{-1}(\kappa/\text{Re } k_{zt})$, is plotted in figure 1 versus (a) angle of incidence θ for the relative speeds $\beta \in \{-0.5, 0, 0.7\}$ and (b) relative speed β for the angles of incidence $\theta \in \{0^\circ, 45^\circ, 75^\circ\}$. We see that (a) for a fixed angle of incidence θ , the orientation angle of $\text{Re } \mathbf{k}_t$ increases as β increases (with the exception of the case of normal incidence wherein the orientation angle of $\text{Re } \mathbf{k}_t$ is 0° for all values of β); and (b) for a fixed relative speed β , the orientation angle of $\text{Re } \mathbf{k}_t$ increases as θ increases. Significantly, $\tan^{-1}(\kappa/\text{Re } k_{zt}) > 0$ (< 0) for all values of β , provided that $\theta > 0$ (< 0). Thus, we see that refraction at the interface $z = 0$ is always positive, regardless of the values of β and θ .

Next we turn to energy flux in the half-space $z > 0$, as provided by the cycle-averaged Poynting vector \mathbf{P}_t . In the limit $|\mathbf{r}| \rightarrow 0$, we have [6]

$$\mathbf{P}_t|_{\mathbf{r}=0} = \frac{1}{2} [(\text{Re } \mathbf{e}_t) \times (\text{Re } \mathbf{h}_t) + (\text{Im } \mathbf{e}_t) \times (\text{Im } \mathbf{h}_t)]. \tag{16}$$

The case of incident *s*-polarization state for which

$$\mathbf{e}_i = a_s \mathbf{e}_\perp \equiv a_s \hat{\mathbf{y}} \tag{17}$$

is distinguished from the case of the incident *p*-polarization state described by

$$\mathbf{e}_i = a_p \mathbf{e}_\parallel \equiv a_p (-\cos \theta \hat{\mathbf{x}} + \sin \theta \hat{\mathbf{z}}), \tag{18}$$

with $a_{s,p}$ being complex-valued amplitudes. In figure 2, the orientation angle of the cycle-averaged Poynting vector \mathbf{P}_t evaluated in the limit $|\mathbf{r}| \rightarrow 0$, as defined by $\tan^{-1}(\mathbf{P}_t \cdot \hat{\mathbf{x}}/\mathbf{P}_t \cdot \hat{\mathbf{z}})|_{\mathbf{r}=0}$, is plotted versus (a) the angle of incidence θ for the relative speeds $\beta \in \{-0.5, 0, 0.7\}$ and (b) the relative speed β for the angles of incidence $\theta \in \{0^\circ, 45^\circ, 75^\circ\}$. Also presented in this figure are the corresponding plots of the quantity $\mathbf{P}_t|_{\mathbf{r}=0} \cdot \text{Re } \mathbf{k}_t$ which

⁴ The corresponding relative permittivity in the inertial reference frame Σ does not feature in our theoretical approach; instead the material response in Σ is specified via the Lorentz-transformed field phasors \mathbf{D} , \mathbf{E} , \mathbf{B} and \mathbf{H} , per equation (2), at angular frequency ω , per equation (8)₃.

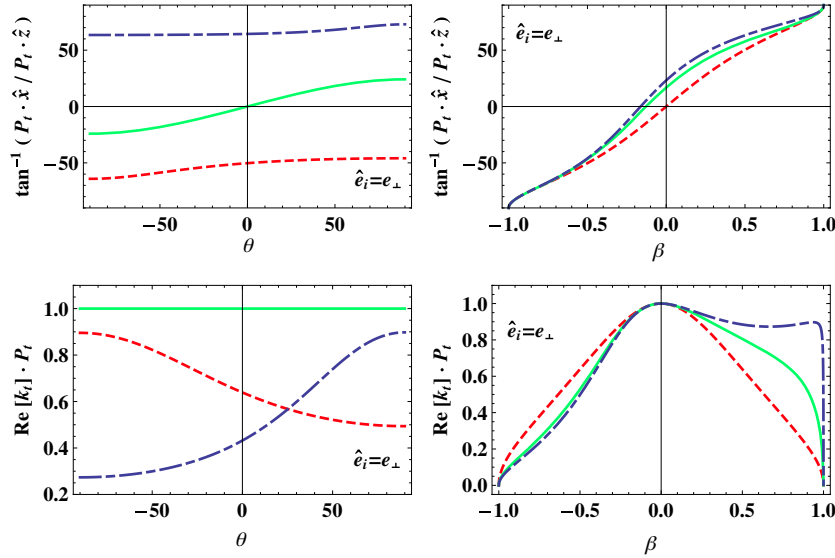


Figure 2. The orientation angle of the cycle-averaged Poynting vector \mathbf{P}_t and the quantity $\mathbf{P}_t \cdot \text{Re } \mathbf{k}_r$ (normalized) plotted against angle of incidence $\theta \in (-90^\circ, 90^\circ)$ for $\beta = -0.5$ (broken curve, red), $\beta = 0$ (solid curve, green) and $\beta = 0.7$ (broken dashed curve, blue); and plotted against relative speed $\beta \in (-1, 1)$ for $\theta = 0^\circ$ (broken curve, red), $\theta = 45^\circ$ (solid curve, green) and $\theta = 75^\circ$ (broken dashed curve, blue). The refracting half-space is occupied by material A ($\epsilon' = 6 + 0.05i$). Plots are shown for the incident s -polarization state. (Colour online.)

determines whether the phase velocity is positive or negative. The graphs in figure 2 are for the incident plane wave having the s -polarization state; the corresponding graphs for the p -polarized incident plane wave are almost (but not exactly) identical. The orientation angle of $\mathbf{P}_t|_{r=0}$ is observed to (a) increase as β increases, at a fixed angle of incidence and (b) increase as θ increases, at a fixed relative speed. Furthermore, by comparing figures 1 and 2, it is apparent that counterposition of $\mathbf{P}_t|_{r=0}$ and $\text{Re } \mathbf{k}_r$ occurs when $-\beta$ is sufficiently large for $\theta > 0$, and when β is sufficiently large for $\theta < 0$. Counterposition does not occur at all when $\beta = 0$, regardless of the angle of incidence. In addition, the phase velocity of the refracted plane wave is positive for all values of β and θ .

Let us now consider a second numerical example where the dissipative material occupying $z > 0$ possesses the relative permittivity $\epsilon' = -0.34 + 0.04i$ at angular frequency ω' in the inertial reference frame Σ' . This material, which we call material B, has the relative permittivity of aluminium at the free-space wavelength $\lambda_0 = 103 \text{ nm}$ [17]. Material B is radically different from material A insofar as the former can support NPV propagation at rest, as we demonstrate in the following.

The orientation angle of the real part of \mathbf{k}_r is plotted in figure 3 versus (a) the angle of incidence θ for the relative speeds $\beta \in \{-0.5, 0, 0.7\}$ and (b) the relative speed β for the angles of incidence $\theta \in \{0^\circ, 45^\circ, 75^\circ\}$. We see that (a) the orientation angle of $\text{Re } \mathbf{k}_r$ increases uniformly as θ increases, at approximately the same rate regardless of the value of β and (b) for $90^\circ > \theta \gtrsim 10^\circ$, the orientation angle of $\text{Re } \mathbf{k}_r$ is nearly 90° , and for $-90^\circ < \theta \lesssim -10^\circ$, the orientation angle of $\text{Re } \mathbf{k}_r$ is nearly -90° , regardless of the value of β . And, in the limits $\beta \rightarrow \pm 1$, the orientation angle of $\text{Re } \mathbf{k}_r$ tends to 0° . Notably, the orientation angle of $\text{Re } \mathbf{k}_r$

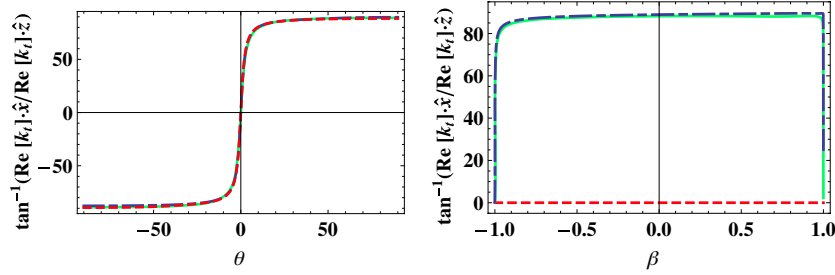


Figure 3. As figure 1 except that material A is replaced by material B ($\epsilon' = -0.34 + 0.04i$).

is always 0° for the case of normal incidence, at all values of $\beta \in (-1, 1)$. As in the previous example, refraction at the interface $z = 0$ is positive for all values of β and θ .

Graphs of the orientation angle of the cycle-averaged Poynting vector evaluated in the limit $|\mathbf{r}| \rightarrow 0$, analogous to those in figure 2, are presented in figure 4. Unlike the case for material A, graphs for the incident s - and p -polarization states are quite different here. The orientation angle of $\mathbf{P}_t|_{\mathbf{r}=\mathbf{0}}$ is observed to be nearly $\pm 90^\circ$ for most values of $\beta \in (-1, 1)$ and for $\theta \in (-90^\circ, 90^\circ)$. There exist (a) a narrow range of values of θ for a fixed value of β (that is centred on $\theta = 0^\circ$ for $\beta = 0$) and (b) a narrow range of values of β (that is centred on $\beta = 0$ for $\theta = 0^\circ$) for which the orientation angle of $\mathbf{P}_t|_{\mathbf{r}=\mathbf{0}}$ passes through 0° , as it increases (or decreases) from nearly -90° (or 90°) to nearly 90° (or -90°). Counterposition of $\mathbf{P}_t|_{\mathbf{r}=\mathbf{0}}$ and $\text{Re} \mathbf{k}_t$ occurs at $\beta = 0$ for the incident p -polarization state but not for the incident s -polarization state, for all angles of incidence except $\theta = 0^\circ$ (when $\mathbf{P}_t|_{\mathbf{r}=\mathbf{0}}$ and $\text{Re} \mathbf{k}_t$ are both directed normally away from the interface). For the incident s -polarization state, counterposition occurs when $\beta > 0$ for $\theta = 0^\circ$, $\beta > 0.6$ for $\theta = 45^\circ$ and $\beta > 0.92$ for $\theta = 75^\circ$. For the incident p -polarization state, counterposition occurs when $\beta < 0$ for $\theta = 0^\circ$, $\beta < 0.6$ for $\theta = 45^\circ$ and $\beta < 0.92$ for $\theta = 75^\circ$.

The sign of the phase velocity can be inferred from the corresponding plots $\mathbf{P}_t|_{\mathbf{r}=\mathbf{0}} \cdot \text{Re} \mathbf{k}_t$ which are also presented in figure 4. The phase velocity of the refracted plane wave is positive at $\beta = 0$ for all angles of incidence, when the incident plane wave is s polarized. However, when the incident plane wave is p polarized, the refracted plane wave has NPV at $\beta = 0$ for $\theta < -3^\circ$ and $\theta > 3^\circ$. For the incident s -polarization state, NPV arises in the refracted field when $\beta > 0.6$ for $\theta = 45^\circ$ and $\beta > 0.92$ for $\theta = 75^\circ$. For the incident p -polarization state, NPV arises in the refracted field when $\beta < 0.6$ for $\theta = 45^\circ$ and $\beta < 0.92$ for $\theta = 75^\circ$.

3. Beam propagation through a moving slab

3.1. Theoretical considerations

Suppose now that the uniformly moving material which occupied the half-space $z > 0$ in section 2 is restricted in its extent to a slab region occupying $0 < z < L$. As before, the uniformly moving material is described by the constitutive relations 1 in the inertial reference frame Σ' . The regions $z > L$ and $z < 0$ are vacuous.

With respect to the inertial reference frame Σ , a two-dimensional beam with electric field phasor [18]

$$\mathbf{E}_i = \int_{-\infty}^{\infty} \mathbf{e}_i(\psi) \Psi(\psi) \exp[i(\mathbf{k}_i \cdot \mathbf{r} - \omega t)] d\psi, \quad z \leq 0 \quad (19)$$

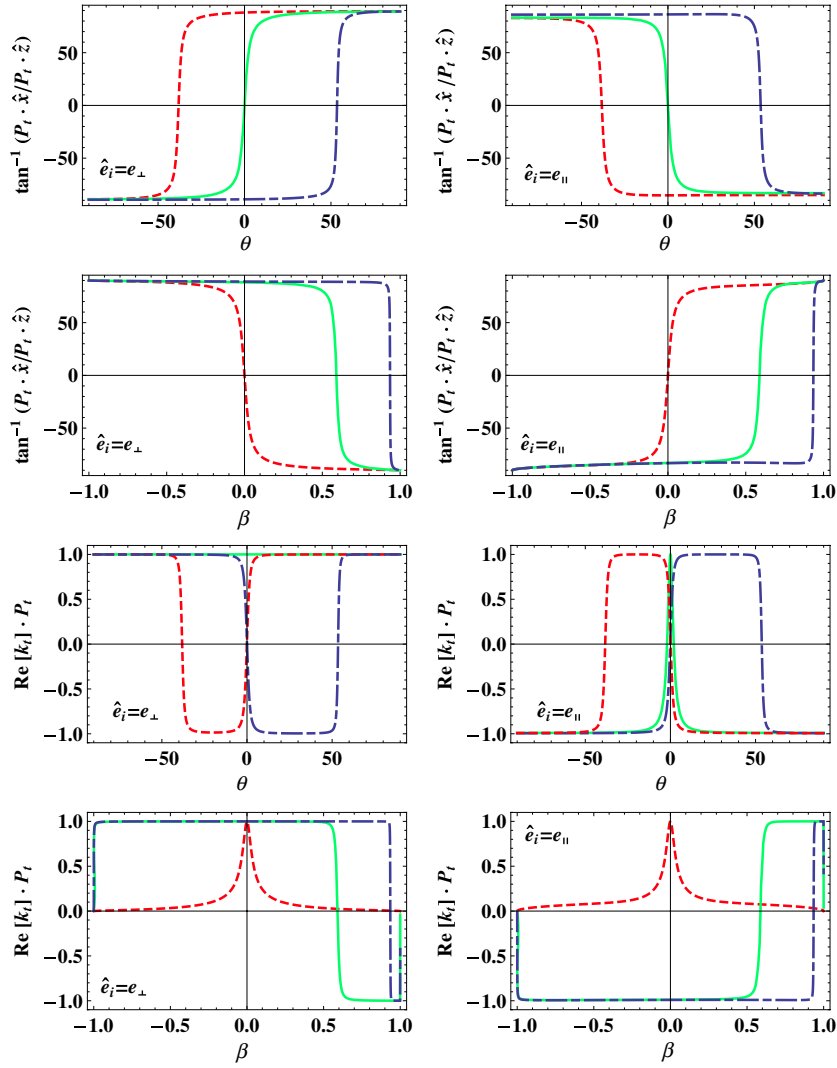


Figure 4. As figure 2 except that material A is replaced by material B ($\epsilon' = -0.34 + 0.04i$), and here the plots for the incident s -polarization state (left) are quite different to the corresponding plots for the incident p -polarization state (right).

is incident upon the uniformly moving slab at a mean angle θ relative to the slab's thickness direction \hat{z} . The beam is an angular spectrum of plane waves, with each planewave contributor having the wave vector

$$\mathbf{k}_i = k_0[(\psi \cos \theta + \sqrt{1 - \psi^2} \sin \theta)\hat{x} - (\psi \sin \theta - \sqrt{1 - \psi^2} \cos \theta)\hat{z}]. \quad (20)$$

A Gaussian form

$$\Psi(\psi) = \frac{k_0 w_0}{\sqrt{2\pi}} \exp\left[-\frac{(k_0 w_0 \psi)^2}{2}\right] \quad (21)$$

is adopted for the angular-spectral function $\Psi(\psi)$, with w_0 being the width of the beam waist [18]. Two polarization states are considered: the s -polarization state described by (17) and the p -polarization state for which

$$\mathbf{e}_i(\psi) = a_p \mathbf{e}_\parallel \equiv a_p [(\psi \sin \theta - \sqrt{1 - \psi^2} \cos \theta) \hat{\mathbf{x}} + (\psi \cos \theta + \sqrt{1 - \psi^2} \sin \theta) \hat{\mathbf{z}}]. \quad (22)$$

As the velocity is tangential to the interfaces and lies wholly in the plane of incidence, the reflected and transmitted beams have spatial Fourier representations similar to that of the incident beam given by (19). In Σ , the reflected beam is represented by the electric field phasor

$$\mathbf{E}_r = \int_{-\infty}^{\infty} \mathbf{e}_r(\psi) \Psi(\psi) \exp[i(\mathbf{k}_r \cdot \mathbf{r} - \omega t)] d\psi, \quad z \leq 0, \quad (23)$$

with the wave vector of each planewave contributor being

$$\mathbf{k}_r = k_0 [(\psi \cos \theta + \sqrt{1 - \psi^2} \sin \theta) \hat{\mathbf{x}} + (\psi \sin \theta - \sqrt{1 - \psi^2} \cos \theta) \hat{\mathbf{z}}] \quad (24)$$

and the corresponding amplitudes

$$\mathbf{e}_r(\psi) = \begin{cases} r_s \mathbf{e}_\perp & \text{for } \mathbf{e}_i(\psi) = \mathbf{e}_\perp, \\ r_p [-(\psi \sin \theta - \sqrt{1 - \psi^2} \cos \theta) \hat{\mathbf{x}} \\ + (\psi \cos \theta + \sqrt{1 - \psi^2} \sin \theta) \hat{\mathbf{z}}] & \text{for } \mathbf{e}_i(\psi) = \mathbf{e}_\parallel. \end{cases} \quad (25)$$

Likewise, in Σ , the transmitted beam is represented by the electric field phasor

$$\mathbf{E}_\tau = \int_{-\infty}^{\infty} \mathbf{e}_\tau(\psi) \Psi(\psi) \exp[i\mathbf{k}_\tau \cdot (\mathbf{r} - L\hat{\mathbf{z}}) - \omega t] d\psi, \quad z \geq L, \quad (26)$$

with $\mathbf{k}_\tau = \mathbf{k}_i$ and the amplitudes

$$\mathbf{e}_\tau(\psi) = \begin{cases} \tau_s \mathbf{e}_\perp & \text{for } \mathbf{e}_i(\psi) = \mathbf{e}_\perp, \\ \tau_p \mathbf{e}_i(\psi) & \text{for } \mathbf{e}_i(\psi) = \mathbf{e}_\parallel. \end{cases} \quad (27)$$

The electric field phasors for the reflected and transmitted beams in Σ are calculated via the strategy outlined in section 2. That is, first the problem is transformed to the inertial reference frame Σ' wherein the boundary-value problem is solved for each planewave contributor to the incident beam, using expressions for the reflection coefficients $r_{s,p}$ and transmission coefficients $\tau_{s,p}$ appropriate for Σ' [11]. Then the electric phasors for the contributor plane waves are transformed back to Σ and integrated with respect to ψ to yield $\mathbf{E}_{r,\tau}$.

3.2. Numerical studies

The slab thickness was fixed at $L = 3\lambda_0$. Following section 2.2, we begin by considering a slab made of material A ($\epsilon' = 6 + 0.05i$). In figure 5, the reflectances $|r_s/a_s|^2$ and transmittances $|\tau_s/a_s|^2$ are plotted against the angle of incidence $\theta \in (-90^\circ, 90^\circ)$, for the relative speeds $\beta \in \{-0.5, 0, 0.7\}$. The reflectances at the angles of incidence θ and $-\theta$ are the same for $\beta = 0$, but this is not true for $\beta \neq 0$; and the same is true for the transmittances. The reflectances and transmittances oscillate markedly as θ increases. This effect—which is due to multiple reflections at the $z = 0$ and $z = L$ interfaces—disappears as the slab thickness increases (as we confirmed in calculations not displayed here). The corresponding reflectances and transmittances for the incident p -polarization state (i.e., $|r_p/a_p|^2$ transmittances $|\tau_p/a_p|^2$) are qualitatively similar to those presented in figure 5.

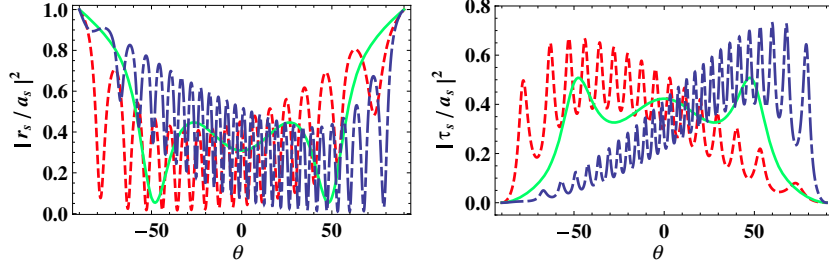


Figure 5. The reflectances $|r_s/a_s|^2$ and transmittances $|\tau_s/a_s|^2$, plotted against angle of incidence $\theta \in (-90^\circ, 90^\circ)$, for $\beta = -0.5$ (broken curve, red), $\beta = 0$ (solid curve, green) and $\beta = 0.7$ (broken dashed curve, blue). The moving slab of thickness $L = 3\lambda_0$ is made of material A ($\epsilon' = 6 + 0.05i$). (Colour online.)

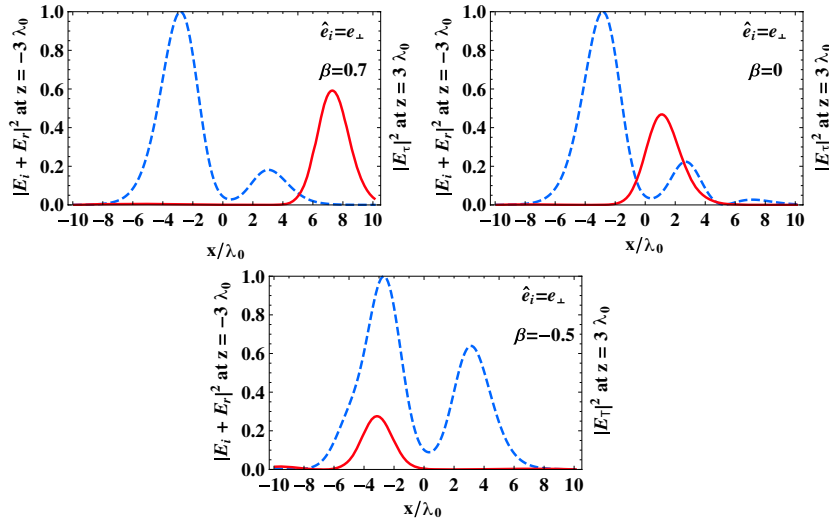


Figure 6. The normalized energy densities at $z = 3\lambda_0$ (solid curves, red) and $z = -3\lambda_0$ (broken curves, blue) plotted against $x \in (-10\lambda_0, 10\lambda_0)$ for $\beta = 0.7, 0$ and -0.5 , for an incident s -polarized beam. The moving slab of thickness $L = 3\lambda_0$ is made of material A ($\epsilon' = 6 + 0.05i$). The centre of the incident beam impinges on the moving slab at the point $x = 0, z = 0$, at an angle of incidence $\theta = 45^\circ$. (Colour online.)

For a mean angle of incidence $\theta = 45^\circ$, with the incident beam focussed on $\{x = 0, z = 0\}$ with waist $w_0 = \lambda_0$, we investigated the reflected and transmitted beams at the relative speeds $\beta \in \{-0.5, 0, 0.7\}$. The energy density

$$|\mathbf{E}|^2 = \begin{cases} |\mathbf{E}_i + \mathbf{E}_r|^2 & \text{for } z \leq 0, \\ |\mathbf{E}_t|^2 & \text{for } z \geq L. \end{cases} \quad (28)$$

was used for this purpose.

The beam energy densities at the planes $z = -L$ and $z = L$ are plotted versus $x \in (-10\lambda_0, 10\lambda_0)$ in figure 6 for the incident s -polarization state. The corresponding graphs for the incident p -polarization state are broadly similar. As β decreases, the energy density of the transmitted beam is concentrated at lower values of x . Indeed, the transmitted beam

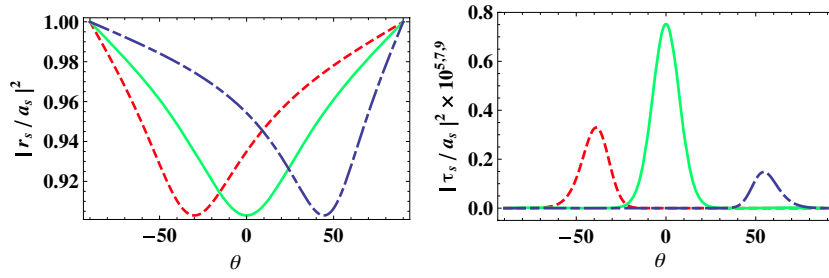


Figure 7. As figure 5 except that material A is replaced by material B ($\epsilon' = -0.34 + 0.04i$), and here the transmittances are multiplied by 10^7 for $\beta = -0.5$, by 10^9 for $\beta = 0$ and by 10^5 for $\beta = 0.7$.

emerges from the moving slab at locations with $x < 0$ when $\beta = -0.5$. This is in consonance with the orientation of the corresponding cycle-averaged Poynting vectors, as presented in figure 2. We note that the maximum amplitude of the energy density of the transmitted beam decreases as β decreases for the incident s -polarization state (but this is not the case for the incident p -polarization state). Also, the maximum amplitude of the energy density of the reflected beam increases as β decreases for the incident s -polarization state (but decreases as β decreases for the other incident polarization state). Extrapolating from figure 6, we estimate that at $\beta \approx 0.2$ the energy density of the transmitted beam would be concentrated at $x = 3\lambda_0$ on the face $z = L$ of the slab. The beam would be essentially undeflected by the moving slab at this relative speed and thereby a degree of concealment of the slab may be achieved [11].

Now, in view of section 2.2, we turn to the case where the slab is made of material B ($\epsilon' = -0.34 + 0.04i$). All other parameters are the same as those used for figures 5 and 6. The reflectances and transmittances are plotted against the angle of incidence $\theta \in (-90^\circ, 90^\circ)$, for $\beta \in \{-0.5, 0, 0.7\}$ in figure 7. The graphs of the reflectances and transmittances are symmetric about $\theta = 0^\circ$ for $\beta = 0$, but they are asymmetric for $\beta \neq 0$. We note that the magnitudes of the transmittances are very much smaller than those of the reflectances, with the largest transmittances occurring when the relative speed β is largest. The oscillations apparent in figure 5 are largely absent from the graphs of the reflectances and transmittances in figure 7. This is because of the much greater degree of attenuation within the uniformly moving slab in the present case.

The beam energy densities at $z = -L$ and $z = L$ are mapped across $x \in (-10\lambda_0, 10\lambda_0)$ in figure 8 for $\beta \in \{-0.5, 0, 0.7\}$, at a mean angle of incidence $\theta = 45^\circ$. For the incident s -polarization state (and the incident p -polarization state which is not represented in figure 8), the energy density of the transmitted beam—which is very much lower than that of the reflected beam—decreases greatly as the relative speed β decreases. This is in keeping with the behaviours of the corresponding reflectances and transmittances, as presented in figure 7. Furthermore, the energy density of the transmitted beam is largely directed normally, away from the interface $z = L$. This may be explained by considering the direction of the corresponding cycle-averaged Poynting vector, as presented in figure 4. The cycle-averaged Poynting vector is approximately aligned with \hat{z} for an angle of incidence of approximately 55° for $\beta = 0.7$. Thus, the maximums of the transmittances at $\theta \approx 55^\circ$ for $\beta = 0.7$ give rise to the maximums in the transmitted energy density in the direction of \hat{z} for $\beta = 0.7$ observed in figure 8. And similarly for the instances where $\beta = 0$ and $\beta = -0.5$, but here the energy densities of the transmitted beams are much lower.

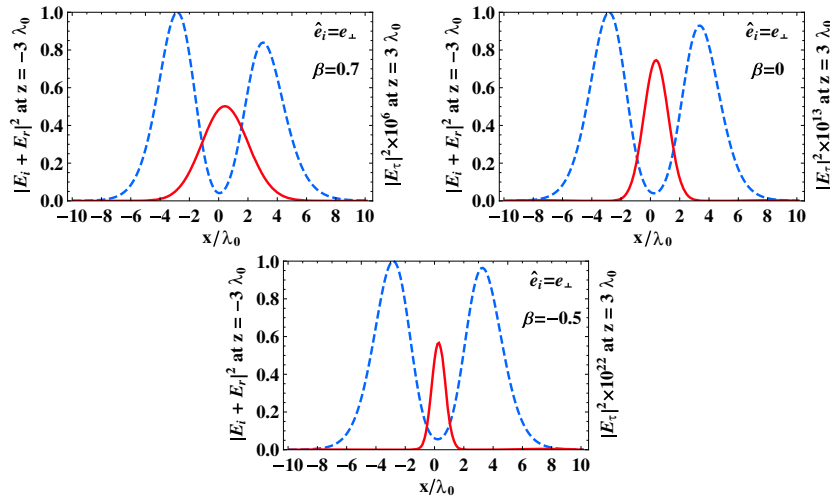


Figure 8. As figure 6 except that material A is replaced by material B ($\epsilon' = -0.34 + 0.04i$).

4. Concluding remarks

By directly implementing the Lorentz transformations of the electric and magnetic fields, we have confirmed that counterposition and NPV are not Lorentz-covariant phenomena, in the context of dissipative mediums. This finding had previously been established for nondissipative mediums using the Minkowski constitutive relations [9, 11]. We further demonstrated that the lateral position of a Gaussian beam, upon traversal through a uniformly moving slab, as perceived by a non-co-moving observer, could be controlled by varying the slab's velocity, when the velocity is tangential to the interfaces and lies wholly in the plane of incidence. Two instances are especially noteworthy: (i) at certain velocities, the transmitted beam can emerge from the slab laterally displaced in the direction opposite to the direction of incident beam and (ii) at a unique slab velocity, the beam can emerge from the slab with no lateral shift in its position, thereby achieving a degree of concealment for the slab [11]. For a uniformly moving slab made of a material which supports both positive and negative phase velocity when at rest (material B in the previous sections), it was seen that the transmittances are very small and the energy density of the transmitted beam is largely concentrated in the direction normal to the slab, regardless of the inertial frame of reference.

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